Department of Mathematics

Pattamundai College, Pattamundai

5th Semester

Probability and Statistics

Core -12

Sec - A Unit - I

1. What is discrete sample space, with one example.
2. Write a sample space which contains 12 elements.
3. Porve that $P(A^c) = 1 - P(A)$ by using probability axiom.
4. If A and B are two independent event then A and B are independent or dependent.
5. Show that, if A and B are two independent events then P(A/B) = P (A)
6. If x and y are two random variable then 5X-7Y is a random variable.
7. The range of a random variable is called the spectrom of the random variable. (T/F)
8. Write the domain and range of a distribution function.
9. The set of points of discontinuity of a distribution function is countable. (T/F)
10. If the pdf of a random variable is

$$f(x) = \begin{cases} k\,x^2 & 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$. Then find the value of K.

11. If x has the pdf $f(x) = \begin{cases} \frac{1}{6} & 0 < x < 3 \\ 0 & \text{otherwise} \end{cases}$

Find the expected value of $y = g(x) = x^3 + 2x$

12. If C_1, C_2, C_3 .... C_n are constants and $g_1(x), g_2(x)$,........ $g_n(x)$ are any functions of x then

$E\left[\sum_{i=1}^{n} c_i g_i(x)\right] =$ _______ ?

Unit - 2

13. What is marginal distribution function ?
14. The joint pdf of (x,y) is

$$f(x,y) = \begin{cases} c(x+2y), & x = 1, 2, 3,\ y = 0, 1, 2 \\ 0 & \text{else where} \end{cases}$$ Then find the value of c.

P.T.O.

15. Let the joint pdf of (x,y) is

$$f(x,y) = \begin{cases} \frac{1}{4}\,(1 + xy), & |x| < l;\ |y| < 1 \\ 0 & \text{else where} \end{cases}$$

Then find $f_X^{(x)}$?

16. Let (x,y) be a two dimensional random variable, then x any y are defined to be stochastically independent If $f_{x,y}(x,y) = f_x^{(x)}.\ f_y^{(y)}$. for all $(x,y)R^2$. (T/F).

17. Write the mean and variance of uniform distribution.

18. Write the neam of a distribution where the mean and variance are equal.

19. What is the difference between the binomial distribution and Negative Binomial distribution .

20. Write the probability distribution for negative binomial distribution.

21. What is the moment generating function of geometric distribution.

22. The joint pdf of $f(x,y) = \begin{cases} 4xy, & 0<x<1\ ;\ 0<y<1 \\ 0 & \text{else where} \end{cases}$

Then find $P(o<x< \frac{1}{3}, \frac{1}{2} < y < 1)$

23. If the joint pdf of (x,y) is

$$f(x,y) = \begin{cases} \frac{3}{16}\,(4-2x-y), & x>0,\ y>0,\ 2x+y<4 \\ 0 & \text{else where} \end{cases}$$

Then find $f_{y/x}(y/x)$.

24. Write the mean and variance of negative Binomial distribution.

<u>Unit - 3</u>

25. Write the mean and variance of Normal distribution.

26. Write the probability density function of Gamma distribution.

27. Write a distribution function in continuous random variable where the mean and variance are equal.

28. Write the mean and variance of Beta distribution.

29. Write the difference between the normal distribution and standard normal distribution.

30. The graph of normal distribution curve is a bell shaped. (T/F)

31. Write the characteristic function of normal distribution.

32. Write a distribution function where the mean, metian and mode are coincide.

33. In a gamma distribution if $\alpha = 1$ and $\beta = \frac{1}{\lambda}$, then it convert to _______ distribution.

34. In a Beta distribution if α= 1 and β =1, then it convert to ______ distribution.

35. Show that $\phi(-z) = 1 - \phi(z), z > 0$

36. Write the joint probability dencity function of a bivariate normal distribution.

UNIT - 4

37. If x and y are two random variable with cov (x,y) = 0 then x and y are independent. (T/F)

38. If x and y are independent random variables with means 4 and 5 and variance 1 and 2 respectively then var(z), where z = 3x – 2y.

39. $E(x/y = y) = \int_{-\infty}^{\infty} x.f(x/y)\,dx$, for continuous case. (T/F).

40. Let x and y be two random variables with the pdf f(x,y), of $f(x,y) = f_X^{(x)}.f_y^{(y)}$ then x and y are independent random variable ? (T/F)

41. Write the moment generating function of Binomial distribution.

42. $E(x^r)$ = coefficient of $\left(\frac{t^r}{r\,!}\right)$ in $Mx^{(t)}$. (T/F)

43. If C is any constant. $Mcx^{(t)} = Mx^{(ct)}$. (T/F)

44. Define the covariance of x and y.

45. If r = 1, the correlation is perfect and positive. (T/F)

46. Write the spearmans formula for the rank correlation coefficient.

47. If r = $\pm$1 and θ = 0 or π . In this case the two lines of regression either coincide or they are parallel to each other. (T/F).

48. Suppose that in the bivariate distribution (x_i, y_i) i = 1, 2, 3 .... n; y is dependent variable and x is independent variable. Let the line of regression of y on x be ______.

49. If the lines of regression are $y = \frac{1}{4}x$ and $x = \frac{1}{9}y + 1$ then & = $\frac{1}{6}$ and E (x/y=0) = 1 (T/F)

UNIT - 5

50. Let x_n be a sequence of random variables such that $x_n \xrightarrow{L} X$ then $x_n + c \xrightarrow{L} X+c$, where c is any constant. (T/F)

51. Write the difference between the weak and strong law of large number.

52. Let $\{x_n, y_n\}$, n = 1, 2, 3 .... be a sequence of pairs of random variables and $x_n \xrightarrow{L} x$, $y_n \xrightarrow{p} c$ then

$x_n \pm y_n \xrightarrow{L} x \pm c$ (T/F)

53. Write only the statement of markovs in equality.

54. Let $x_n \xrightarrow{p} x$ and g be a continuous function on IR. Then $g(x_n) \xrightarrow{p}$ _____ ?

55. Let $x_1, x_2, x_3 .... x_n$ be the i.j.d random variables follow N (μ, σ^2) Then find E $(\bar{x})$ = _____ ?

56. Let $x_1, x_2, x_3 .... x_n$ be te i.j.d random variables follows U (a,b) then find V $(x_1+x_2+x_3 ...... + x_n)$ = ____ ?

57. What is transition probability matrix.

58. Let $A = \begin{pmatrix} 0.1 & 0.9 \\ 0.8 & 0.2 \end{pmatrix}$ is a stochastic matrix.

59. $P_{ij}^{(n)} = p[x_{m+n} = j / x_m = i]$ (T/F)

60. Suppose $A = \begin{pmatrix} \frac{1}{10} & \frac{3}{10} & \frac{6}{10} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{4} & 0 & a \end{pmatrix}$ is a transition probability

matrix, then find the value of a = ________ ?

SEC - B (Unit - 1)

1. If A and B are any two events in S then prove that $p(A \cup B) + p(A \cap B) = p(A) + p(B)$

2. For $(n \geq 2)$ mutually exclusive events A_1, A_2, A_3 .... An i.e.

$A_i \cap A_j = \phi$, $(i \neq j$, i.j = 1, 2, 3 .....n). Then prove that $P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} p(A_i)$

3. If A and B are two independent events then prove the A^c and B^c are also independent.

4. Let a coin be tossed two times in succession. Define the events A, B and C as

A = the event that head appears in the first toss.

B = the event that the head appears in the 2nd toss.

C = the event that only one head appears in a toss. Then prove that A, B, C are not mutually independent.

5. Three players A, B, C toss a coin alternately so that the game is terminated as soon as we get a head is obtained. Find the respective chances of winning the game.

6. If we toss a coin 3 times and x denote the number of heads, then find the probability distribution table for x.

7. Prove that F(x) – F(–x) = P(x=x).

8. The probability density function of random variable x is given by $f(x)=\begin{cases}6x(1-x), & o<x<1\\ 0 & \text{else where}\end{cases}$

Then find F(x)

9. The distribution function of the random variable x is given by

$$f(x)=\begin{cases}1-(1+x)e^{-x}, & x>0\\ 0 & , x\le 0\end{cases}$$

Then find (a) $p(x \le 2)$ (b) p (1<x<3)

10. If x is the number, obtained when a blance die is rolled, find the expected value of $y = g(x) = x^2+2x+1$

11. If C_1, C_2, C_3........ Cn are constants and $g_1(x), g_2(x)$........ $g_n(x)$ are any function of x then prove that

$$E\left(\sum_{i=1}^{n} cigi(x)\right) = \sum_{i=1}^{n} ciE\left(g_i(x)\right)$$

12. Suppose x is any random variable then prov that $E(ax^2+bx+c) = a\,E(x^2)+bE(x)+c$, where a, b, c $\in$ R.

UNIT - 2

13. Let us consider a bivariate pdf in continuous case. If we wish to select a point randomly from in side a circle $x^2+y^2 = 25$. Then find the joint pdf of x and y.

14. If the joint probability density of two random variables is $f(x,y)=\begin{cases}6e^{-2x-3y}, & x,y>o\\ 0 & \text{else where}\end{cases}$

Then show that x and y are independent variable.

15. Let x and y have the joint pdf

$$f(x,y)=\begin{cases}x+y, & o<x<1,\ o<y<1\\ 0 & \text{else where}\end{cases}$$

Then find $f_{x/y}^{(x/y)}$ and $f_{y/x}^{(y/x)}$

16. The probability density function of a two dimensional random variable (x,y) is

$$f(x,y) = \begin{cases} \frac{1}{8}(x+y), & 0 \le x, y \le 2 \\ 0 & \text{else where} \end{cases}$$

The evaluate p(lx–yl>l)

17. A perfect die is thrown 8 times . Find the probability of getting atmost 3 times 5 or 6 as the outcome.
18. If the number of accidents occuring on a highway each day is a poisson random variable with parameter α = 4, what is the probability that no accidents occur to day.
19. If a boy is throwing at a target, what is the probability that his 12^{th} throw is his 7^{th} hit if the probability of hitting that target is 0.4 ?
20. If a blanced dia is thrown, find the probability that a '6' first appears on the 7^{th} trial.
21. With the usual notations, find p for a binomial variate x if n = 6 and 9 p (x = 4) = p (x=2)
22. The mean and variance of binomial distribution are 4 and $\frac{4}{3}$ respectively. Find p $(x \geq 1)$
23. If x is a poisson variate such that

    p (x=2) = 9 p(x=4) + 90 p(x=6)

    Then find the value of λ.
24. Find the mean of the binomial distribution

$$\binom{10}{x}\left(\frac{2}{3}\right)^x\left(\frac{3}{5}\right)^{10-x}, \quad x = 0, 1, 2......10$$

UNIT - 3

25. A random variable $x \sim \cup (-3, 3)$ compute

    a) p(x<1) (b) p(lx–2l<l)
26. In a certain city, the daily consumption of electric power, in million of killowalt hours (Mkh) may be regarded as a random variable having gamma distribution with α = 3 and β = 2. If the power plant has a daily capacity of 12 Mkh, what is the probability that this power supply will be inadequate on any given day?
27. The life time of an electric bulb is a random variable having exponential distribution with λ=0.02. Find the prbailities that such bulb will last.

    a) atleast 130 days.

    b) at most 300 days .
28. If the annual proportion of a component that fail in a certain brand of T.V. set may be looked upon as a random variable having a beta distribution with α = 2 and β = 4. Find the probability that atleast 30% of all components will fail in the television sets of that brand.
29. If x is n(75, 100), then find p(x < 50).

30. Let x be a normal random variable with μ = 5 and σ = 10
Then find
a) p(x>ll) (b) p(6<x<9)

31. Let x ~ n (μ, σ^2) so that p(x<89) = 0.9 and p(x<94) = 0.95. Find and μ σ^2.

32. If 8% of the probability for a certain distribution that n(μ, σ^2) is below 50 and that 5%, is above 90. what is the values of μ and σ^2.

33. Let x ~ rn (75,25) . Find the conditional probability that x is greater than 80 relative to the hypothesis that x is greater than 77.

34. If x ~ Expo(λ) with p(x$\leq$1) = p (x>1). Find the mean and variance of x.

35. Show that the mean and variance of uniform distribution has $\frac{a+b}{2}$ and $\frac{(b-a)^2}{12}$

36. If x ~ n(1,4), y ~ n(2,9) and x and y are independent, find the distribution of u = 2x+3y and hence compute p(u>20).

UNIT - 4

37. Let x_1 and x_2 are two independent random varizbles and a_1, a_2 be constant.

Then v $(a_1x_1+a_2x_2)$ = $a_1^2 v(x_1) + a_2^2 v(x_2)$.

38. If the joint pdf of (x,y) is f(x,y) = $\begin{cases} 24y(1-x), & 0 \le y \le x \le 1 \\ 0 & \text{therwise} \end{cases}$

The find ℓ_{xy}

39. If x and y are two independent random variable have the variance 64 and 25 respectively. Then find suk where u = x+y and v = x – y.

40. Let x_1 and x_2 be two independent random variables

Then show that $M_{x_1+x_2}^{(t)} = M_{x_1}^{(t)}.M_{x_2}^{(t)}$

41. If the MGF of a random variable x is $M_X(t) = \frac{1}{81}(2+e^t)^4$ then find the mean and variance of x.

42. If x and y have joint pdf $f(x,y) = \begin{cases} \frac{1}{40}|x+y|, & x = -2, -1, 0, 1, 2 \\ 0 & , y = -2, -1, 0, 1, 2 \\ & \text{otherwise} \end{cases}$

Then find E(xy)

43. Let x and y have the joint pdf $f(x,y) = \begin{cases} x+y, & o<x<1, o<y<1 \\ 0 & \text{else where} \end{cases}$

Find the conditional pdf of x given y.

44. Find the marginal distribution function of two random variable x any y, whose joint pdf is

$$f(x,y) = \begin{cases} 6e^{-2x-3y} & , x>o, y>o \\ 0 & \text{else where} \end{cases}$$

45. Let the joint pdf of x any y be

$$f(x,y) = \begin{cases} 8xy & 0<x<y<1 \\ 0 & \text{else where} \end{cases}$$

Then find $p(x < \frac{1}{2} n\ y < \frac{1}{4})$

46. Let the pdf of a random variable be

$$f(x) = \begin{cases} \frac{x}{15}, & x = 1, 2, 3, 4, 5 \\ 0 & \text{else where} \end{cases}$$

Then find the MGF of x.

47. If x and y are random variables and a, b, c, d are any numbers provided only that a $\neq$ o, c $\neq$ o, then prove that $r\ (ax+b, cy+d) = \frac{ac}{|ac|} r(x,y)$

48. The variables x and y are connected by the equation ax+by+c= o. Show that the correlation between them is –1 if the signs of a and b are alike and +1, if they are different.

49. Can y = 5 + 2.8x and x = 3–0.5y be the estimated regression equation of y on x and x on y respectively ? Explain your answer with suitable theoretical arguments.

50. Variable x any y have the joint pdf given by $f(x,y) = \begin{cases} \frac{1}{3}(x+y), & o \leq x \leq 1, 0 \leq y \leq 2 \\ 0 & \text{otherwise} \end{cases}$

Then find (i) r (x,y) (ii) The two line of regression.

UNIT - 5

51. Let x_1, x_2, x_3 .... x_n be i.i.d random variables follows u[0,1]. Then prove that $x_n \xrightarrow{p} a$, where a is any constant.

52. Write the sufficient condition for weak law of large numbers.

53. The rainfall x in a certain locality is a normally distributed random variable with mean 40cm and variance 4cm². Find a simple upper band on the probability that rainfall in particular year will exceed the mean by 5cm.

54. Let x be the mean of random sample of size 150 from gamma distribution with $\alpha = 3$ and $\beta = 5$. Then find $p(14 < \bar{x} < 16)$.

55. A random sample of size 100 is taken from a normal population with $\sigma = 25$. What is the probability that the mean of the sample will differ from the mean of the population by 3 or more either way ?

56. Let $x_1, x_2, \ldots\ldots\ldots x_n$ ben random variable which are independent and identically distributed with mean μ and variance σ^2. Then prove that E $(\bar{x})$ = 01 and var $(\bar{x}) = \frac{\sigma^2}{n}$

57. Let $\{x_n\}$ be a sequence of independent and identically distributed random variable with finite mean μ and finite variance σ^2. If $Sn = \sum_{i=1}^{n} xi$, then show that the law of lare number does not hold for the sequencey {Sn}

58. State and prove chapman-kolmogorov equation.

59. Let $\{x_n, n \geq o\}$ be a markov chain with three states 1, 2, 3 with T.P.M. $P = \begin{pmatrix} \frac{1}{2} & \frac{1}{3} & \frac{1}{6} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \\ 0 & \frac{3}{4} & \frac{1}{4} \end{pmatrix}$

Then find graph of T.P.M.

60. Consider the three state markov chain with T.P.M.

$P = \begin{pmatrix} 0 & 1 & 0 \\ 0 & ½ & ½ \\ ½ & 0 & ½ \end{pmatrix}$ with the inital distribution.

$p(x_0 = i) = \frac{1}{3}, i = 0,1,2$. Then find $p(x_3 = 1, x_2 = 1, x_1 = 0, x_0 = 2)$

<u>SEC - C, UNIT - 1</u>

1. Show that for any two events A and B,

$p\ (A \cap B) \leq p\ (A)\ \leq\ p\ (A \cup B)\ \leq\ p\ (A)\ +\ p\ (B)$

2. Four roads lead away from a jail. A prisoner escaping from the jail and selects a road at random of road I is selected the probability of escaping is $\frac{1}{8}$, for road II, it is $\frac{1}{6}$, for road III it is $\frac{1}{4}$ and if road IV is selected the probability of escaping is $\frac{9}{10}$.

a) What is the probability that the prisoner will succeed in escaping ?

b) What is the probability that the prisoner escaping from the jail, he was used the road IV ?

3. Let x denote the random variable that is defined as the sum of the numbers in the two dice. Find the space and the probability distribution function of x.

4. Of the density function of x is $f(x)\begin{cases} ce^{-3x}, & 0 < x < \infty \\ 0 & x \leq 0 \end{cases}$

Then find

a) The value of c

b) $p(x > 2)$

c) $p\ (2<x<3/\ x > \frac{5}{2})$

5. a) If x and y are random variables then prove that $E(x+y) = E(x) + E(y)$

b) If x and y are independent random variables then prove that $E(xy) = E(x) . E(y)$

UNIT - 2

6. Find the mean and variance of the binomial distribution.

7. Find the moment generating function of poisson distribution.

8. Trains arrive at a specified station at 15 minute intervals stating from 9a.m. If a passenger arrives at the station in a random time between 9 and 9.30 . Find the probability that he has no wait.

a) less than 7 minutes.

b) atleast 10 minutes for the train.

9. Show that the poisson distribution is a limiting case of binomial distribution.

10. If the probability is 0.75 that a sales person will convinece his customer to shell his product, find the probability that.

a) The eight customer will be the fifth customer convinced by the sales person.

b) The fifth customer will be the tenth customer convinced by the sales person.

UNIT- 3

11. The bivarite normal random variable (x,y) have parameters $\mu_1 = 60$, $\mu_2 = 75$, $\sigma_1 = 6$ and $\sigma_2 = 12$ and $\delta xy = 0.55$. Find the following probabilities.

a) $p(65 \leq x \leq 75)$ (b) $p(71 \leq y \leq 80 / x = 55)$

12. Scores on a certain test, IQ scores appronimately normally distributed with mean $\mu = 100$ and $\sigma = 1.5$. An individual is selected at random. What is the probability that his score x satisfies $120 < x < 130$?

13. Find the mean and variance of Gamma distribution.

14. If the duration of a shower in an island is exponentially distributed with $\lambda = \frac{1}{5}$.

a) Out of 3 showers what is the probability that not more than 2 will last for 10 minutes or more ?

b) A shower will last atleast 2 minutes more given that it has already lasted for 5 minutes.

c) A shower will not last more that 6 minutes more if it has already leasted for 3 minutes.

15. Let x and y have a bivariate normal distribution with parameters $\mu 1=5, \mu 2=10, \sigma_1^2=1, \sigma_2^2=25$ and $\delta_{xy}>0$

If p(4<y<16/x=5) = 0.954 then find δ_{xy}

UNIT - 4

16. If x_1 and x_2 are two random variables a_1, a_2, b_1, b_2 are constants then prove that

Cov $(a_1x1+a_2x2, b_1x1+b_2x_2) = a_1b_1\, v(x_1) + a_2b_2 v(x_2) + (a_1b_2+a_2b_1)\operatorname{cov}(x_1.x_2)$

17. Let x any y have the joint pdf $f(x,y)=\begin{cases} x+y, & 0<x<1,\ 0<y<1 \\ 0 & \text{other wise}\end{cases}$

Find the conditional mean and variance of y given (x = x)

18. Find the moment generating function of x and find its mean and variance it the pdf is

$$f(x)=\begin{cases} 2\left(\frac{1}{3}\right)^{x} & , x = 1, 2, 3 .... \\ 0 & \text{else where}\end{cases}$$

19. Obtain the equations of two lines of regression for the following data. Also obtain the extimate of x for y = 70.

x :	65	66	67	67	68	60	70	72
y :	67	68	65	68	72	72	69	71

20. The random variables x any y are jointly normally distributed and u, v are defined by u = x cos α + y sin α v = y cos α – x sin α . Show that U and V will be uncorrelated if

$$\tan 2x = \frac{2r\sigma_x\sigma_y}{\sigma x^2-\sigma y^2}$$

Where r = corr (x,y). Are U and V independent ?

UNIT - 5

21. State and prove chebyshevs inequality.

22. Let {xi, i >, 1} be sequency of i.id random variables follows N(2,5) Then prove that

i) $\frac{1}{n}\sum_{i=1}^{n} xi \xrightarrow{p} 2$ (ii) $\frac{1}{n}\sum_{i=1}^{n} x_i^2 \xrightarrow{p} 9$

iii) $\left(\frac{1}{n}\sum_{i=1}^{n} x_i\right)^2 \xrightarrow{p} 4$ (iv) $\sum_{i=1}^{n}\left(\frac{xi}{n}\right)^2 \xrightarrow{p} 0$

23. Let x_1, x_2, x_3 .... x_n be a sequence of independent random variables with $E(xi) = \mu$ and $v(xi) = \sigma^2$.

Let . $\bar{x}n = \frac{\sum_{i=1}^{n} xi}{n}$ Then prove that for any $\in > o$, $p(|\bar{x}n - \mu| > \in) \xrightarrow{p} o$, as $n \to \infty$

24. Find an approximate probability that the mean of random sample of size 20 from a distribution having pdf

$$f(x) = \begin{cases} 4x^3 & , \ o < x < 1 \\ o & \text{else where} \end{cases}$$

is between 0.73 and 0.83

25. A man has four pairs of socks which he changes every day, choosing at random one of the three not worn the previous day. Each day the man has a probability ½ of forgetting to change his socks. Then find the transition probability matrix.

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